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Local government Formula Funding in England since 2013/14: Lessons for future funding models

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Abstract

Since 2013/14, local government in England has received funding allocations on the basis of a set of funding formulas that were last recalculated around 2012, using data from that period and earlier. We have reproduced the funding allocation mechanism in a different format (the computer language R), and used our reimplementation to examine various features of the mechanism. We identify various problems with the allocation model and find that partially updating the data in the model results in substantial deviations from the funding currently allocated under the model. Drawing lessons from the problems identified in the 2013/14 allocation mechanism, a set of criteria are proposed for the behaviour of any future mechanism, and tests are described to support assessments of whether these criteria are being met.

1 Introduction

Funding formulas have been used for distributing large sums of public money for many years. These formulas have faced criticisms on a number of grounds, including from scholars who have identified problems in the way they have been implemented that produce incongrous results. However, responses to these criticisms have sometimes viewed them as "rather more convincing in saying what not to do than in proposing suggestions about what one should do" (Senn in "Discussion on the Meeting on 'Resource Allocation Models"' 2011). We contend that the problems of funding formula mechanisms adopted in the past contain the seeds of useful lessons that can be learnt for future allocation schemes. As detailed below, we have rebuilt the funding formula model in a way that allows us to calculate its outcomes under various alternative scenarios; this scenario testing and related analyses (amongst other factors) allows us to find problems and to verify the continuing existence of problems that have previously been identified. Identified problems (or their inverses) can be formalised into criteria. And tests can be defined to help to detect whether a given funding mechanism meets the criteria or not. Adopting these tests could ensure that future schemes do not repeat the mistakes of the past.

Local government funding in England takes place in a context of a relatively highly centralised system. Under various guises (including the Office of the Deputy Prime Minister (2002-2006), the Department for Communities and Local Government (2006-2018), and Ministry of Housing, Communities and Local Government (2018-)), the central government department responsible for local government issues has devised funding allocation schemes. The core empirical focus of this paper is analysis of the 2013/14 funding formula system for England. It is worth noting that this formula remains directly relevant at the point of writing since it is effectively still in force; the funding allocations from 2014/15 up to and including the 2018/19 allocation have been based on 2013/14 allocations, subject to proportional decreases as government has decreased the overall local government funding envelope. The conclusions we draw and recommendations we make, however, are **not** exclusively linked to that formula, and will have relevance to any future funding allocation mechanisms.

Broadly speaking, the allocation mechanisms adopted by central government have been intended to achieve (partial) **equalisation**, where (full) equalisation is "the principle of compensating local authorities in full for differences in their spending needs and in their [revenue-raising capacity]" (after Jackman, 1985, cited in Sandford 2017, p247). The most recent funding system, in place since 2013, has placed the equalisation principle in tension with an **incentive** objective. Whilst the funding allocation for 2013/14 was based on a system that was *intended to* represent a comprehensive assessment of needs, of the type that can trace its history in England back to 1958 (Sandford 2017, p247), the resources that each authority has had available in subsequent years was designed to deviate from that assessment in line with local growth (or shrinkage) in business rates. This mechanism is intended to "provide a strong incentive for local authorities to promote growth" in their local economies (Department for Communities and Local Government 2012, para. 3).[1] The

[1] Whilst creating a strong growth incentive was the stated aim of the policy, it is worth noting that the design of the system is actually to incentivise growth of the business rates tax base, which may not necessarily generate economic growth (National Audit Office 2017 p7). Stakeholders have expressed scepticism about whether it has incentivised pro-economic growth behaviours (National Audit Office 2017 p31).

shift from equalisation to incentivisation has not been entirely one-way, however, with certain subsequent changes acting "to reduce the influence of incentives within the overall system" with "[equalisation retaining] its place in the conceptual world of English local government finance" (Sandford 2017 p249). Consequently, the systems for assessing needs across local authorities remain an important component of funding allocation.

Previous scholars have examined various details of funding formula mechanisms that have been used to allocate resources across geographical divisions of public services. Most closely related to the current study is the work of Gibson and Asthana (2011), which examined the four-block model of local government funding in England, in force from 2008. The current 2013-based allocations model bears many similarities to the system from 2008, and as the present paper demonstrates, persists in demonstrating some of its most striking flaws. The analysis presented by Gibson and Asthana presented possibly the most striking finding that exists in the literature on funding formulas: "the Wokingham effect" (pp540-541).[2] Briefly, the effect described is that when Wokingham's data was removed from the allocation pot and the formula was recalculated, rather than simply redistributing the authority's (relatively small) allocation, very large swings — both gains and losses — affected some other authorities, including quite unrelated authorities. This result arose because Wokingham was acting as the threshold authority: elements of the calculation acted to calibrate each authority's values with respect to some lowest value, which came from Wokingham. With Wokingham removed, the calculation mechanism drew on next smallest value, with consequent changes. As well as identifying that particularly notable behaviour, their critique concluded that the funding mechanism was "deeply flawed and generates an inequitable allocation of this major source of local authority revenue"; argued that central government should "revisit its resource allocation methodology as a matter of urgency"; and commented more generally on the existence of "risks ... associated with the development of ever more complex methods of resource equalization" (p529).

Other relevant critiques of the general process of using funding formulas include those that have looked at somewhat similar systems for distribution of healthcare funding in England (for example Galbraith and Stone 2011). Such literature has flagged up the potential for "abuse" of regression-based techniques in funding allocations. One approach that has been identified as being potentially problematic is the situation where future funding decisions are made based upon regression models, where the model is calculated with past expenditure (or service utilisation) being used as the 'dependent variable'. This type of approach will — as an inherent feature of its design — generate models that aim to approximately reproduce the *status quo*; with a limited number of indicators ('explanatory variables') included in the model, the regression model will not perfectly replicate the previous allocations, and an authority's allocation will instead be a value that is (loosely speaking) 'typical' given its characteristics (the values it has for each relevant indicator). This can act, to an extent, to entrench a given pattern of funding as it effectively treats the past funding levels as the target that the new formula model should try to fit; this will particularly happen if there is a particular characteristic that has historically been strongly associated with high or low levels of funding relative to the actual underlying need across authorities[3] — inclusion of such a variable in a regression model will tend to reproduce the past pattern of over- and under-funding.

Another problem that can arise in regression (and some other) models is 'over-fitting'. Over-fitting occurs when a model becomes a very good fit for the data it has been 'trained' on, whilst becoming (much) less good for ascertaining the correct value of the dependent variable for any **new** data (in the case of a funding allocation, the 'new data' might be data for the same authorities for new years, and the 'correct' value is one that reflects the underlying funding need of the area given its new characteristics). As the number of indicators included in a regression model is increased, the potential for overfitting increases; in the extreme case, if the number of variables is equal to (or greater than) the number of observations (i.e., for funding

[2]Importantly, although Gibson and Asthana (2011) were the first to identify the issue, that paper was not the first to put it into the public domain. Their finding was summarised by Stone (2009) (p36), so it could have been known by those responsible for the formula several years before the 2013 version was adopted.

[3]Note that this only requires an association to exist between one or more indicators and the **relative** historic funding levels. It could occur if there were a variable where, for example, all authorities that have a high value for that indicator have been over-funded and those that have a low value for it have been under-funded, with those having a middling value having been funded about right. But, in the case where the overall funding envelope is constrained, it could also occur where all authorities are under-funded compared to the resources they believe would be necessary to provide adequate service to their populations, but where the extent of under-funding has historically been proportional to some variable, such that some classes of authorities are more underfunded than others.

models, the number of indicators being equal to the number of authorities whose funding is being modelled) then regression will produce a model that — spuriously — perfectly reproduces the dependent variable (Smith 2014). When overfitting occurs through increasing the number of explanatory variables (including versions less extreme than the perfect match, which are more plausible in the funding formula situation), the weights on each indicator effectively become progressively more arbitrary even while the resultant model appears to be creating a better and better fit to the dependent variable. The detailed application of particular regression techniques has also been criticised (Galbraith and Stone 2011).

A further issue that has been identified — in critiques of data analysis in general (for example Herndon, Ash, and Pollin 2013), and in relation to this subject in particular (including Hewson et al. 2016) — is the susceptibility of spreadsheets and similar tools to the introduction of errors.

This paper proceeds as follows. In the next section we outline the technical approaches that were adopted in calculating and publishing the 2013/14 funding allocation mechanism. Having provided this background, we structure the paper into several subsequent sections that each outline a problem, error or inconsistency that is found in the funding formula; these sections also introduce various reasonable criteria that the funding formula is in breach of, and tests that any future funding allocation mechanism could be subjected to in order to assess whether the criteria are being met.

2 Overview of method and technical approaches

The 2013/14 Formula Funding system was built using a database package called Oracle Financial Analyser. They were then published as Excel spreadsheet versions, separately covering a Relative Needs Formula (RNF) stage and a Formula Funding (FF) stage (Local Government Finance Formula Grant Distribution Division 2013a, 2013b). The calculation process is also described in a document (Department for Communities and Local Government 2013a), and the data definitions are provided across two documents (Department for Communities and Local Government 2013b, 2013c).

Our approach has been to rebuild the 2013/14 formula funding model in the statistical programming language R (R Core Team 2017). This has allowed us to re-run the formulas with multiple different variants of the data and to observe the funding allocations that result. The process of rebuilding the formula in R, by reference to both the definition documents and the spreadsheets, itself acted as a part of the method for identifying problems with the formula.

In some cases we have sought to provide outline explanations of how the funding formula generates the results that we have identified as problematic. For example, the mechanisms that created the Wokingham effect are described in detail in the previous literature (Gibson and Asthana 2011), and we produce a brief outline in this paper. These are provided principally for the interest and illumination of the reader, rather than being strictly necessary for our analysis and prescriptions: by knowing that a problem exists and understanding the nature of the (sometimes clearly incongruous) results that it produces, we would be able to specify a criterion that seeks to rule it out (and associated test for behaviour of any prospective new allocation scheme), irrespective of whether the mechanism by which the present formula generates such results is fully described or not. In fact, we seek to be as mechanism-neutral as possible in proposing criteria and tests, referring to the effects of the problems rather than the causes. Doing so supports our aim of creating solutions that are as general in nature as possible.

3 Adding resources can result in losses (or "why Wokingham still matters")

As noted above, one of the most striking previous findings from analyses of funding formula allocations was "the Wokingham effect", identified in the 2008/09 system by Gibson and Asthana (2011). Taking this unitary authority out of the equation, returning its (small) allocation to the pot and re-running the calculations

Table 1: Authorities with highest absolute gains from removing Wokingham from calculations. Allocations (and differences) in £millions. Values may not match exactly due to rounding.

Local Authority	Class	Allocation without Wokingham	Original allocation	Allocation difference
GLA - police	GLAPOL	2051.2	1966.6	84.6
Birmingham	MD	697.6	653.9	43.6
Tower Hamlets	ILB	227.5	201.8	25.7
Westminster	ILB	163.4	138.1	25.3
Liverpool	MD	312.5	288.8	23.7
Manchester	MD	344.7	321.6	23.1
Hackney	ILB	216.2	194.0	22.2
Southwark	ILB	211.4	191.6	19.8
Newham	OLB	236.3	217.2	19.0
Lambeth	ILB	198.1	181.1	17.0
Islington	ILB	150.1	133.3	16.8
Camden	ILB	149.4	132.8	16.6
West Midlands Police	POL	532.1	516.6	15.5
Lewisham	ILB	185.2	170.2	15.0
Greenwich	ILB	167.8	153.5	14.3

resulted in some of the remaining authorities allocations varying to extents that were large in both absolute and relative terms. Given the incongruous knock-on effects that were observed when taking one authority out of the equation, assessing whether this problem persists in the current model is an important empirical question.

Using our version of the 2013/14 model, we were able to run the code removing each authority in turn and identifying the extent to which the remaining authorities' allocations were affected. We found that — in line with the 2008/09 formula — removing Wokingham still had the largest effects on the funding allocations that were calculated for other authorities, and that these swings were still large.

Tables 1 and 2 show the top and bottom authorities gaining and losing from the removal of Wokingham. In total, 48 authorities gain **more** than Wokingham's £4.44 million allocation that should notionally be redistributed by this process. Even more striking is the large number of authorities — 307 — that **lose** money (i.e., see a reduced allocation) if Wokingham is removed from the calculations. (The remaining 66 authorities gain between £0 and Wokingham's £4.44 million.)

As was identified by the previous study of this effect (Gibson and Asthana 2011), the reason that Wokingham has such a large effect is because it is the threshold authority for important elements of the calculation — the minimum value against which other authorities are compared. Removing Wokingham means that the next smallest value is used, resulting in large differences in key calculated values.

Stone (2012) has described a criterion in response to the Wokingham effect — which he labelled the 'increment principle':

> "For any partaker X in the principled share-out of a particular public good to a group of partakers, the good allocated to X should not decrease with any increase in the total goods allocated to any sub-group that includes X."

A similar criterion may be found in the distributive justice literature, in the *criterion of resource and population monotonicity*:

> "For the case in which a common resource grows in size, ... resource monotonicity demands that every agent should be at least just as well off as from the fair division of the smaller resource." (Helm and Simonis 2001)

Both of these (appropriately, in our view) permit that a given authority's share may not grow; it would be

Table 2: Authorities with highest absolute losses from removing Wokingham from calculations. Allocations (and differences) in £millions. Values may not match exactly due to rounding.

Local Authority	Class	Allocation without Wokingham	Original allocation	Allocation difference
Surrey	SCFIR	31.6	67.2	-35.6
Hampshire	SCNFIR	103.4	137.0	-33.6
Hertfordshire	SCFIR	115.9	142.0	-26.1
West Sussex	SCFIR	83.2	102.9	-19.7
Oxfordshire	SCFIR	75.7	92.0	-16.3
Leicestershire	SCNFIR	82.7	97.9	-15.2
Essex	SCNFIR	245.7	259.0	-13.3
Buckinghamshire	SCNFIR	22.6	35.1	-12.5
Staffordshire	SCNFIR	153.5	165.9	-12.4
Cambridgeshire	SCNFIR	90.7	102.2	-11.5
Northamptonshire	SCFIR	143.1	154.1	-11.0
Warwickshire	SCFIR	86.1	96.6	-10.5
Cheshire East	UNINFIR	45.7	56.0	-10.2
Wiltshire	UNINFIR	82.5	92.5	-10.0
Thames Valley Police	SPOL	231.1	240.6	-9.5

acceptable for some authorities' shares to grow and others to remain constant at the level they held before the authority was removed, though of course none should shrink with an overall growing pot. The breach of these principles in this instance — the 'removed authority' situation — is observed in a scenario where the available resource pot for the remaining authorities would grow. However, in drawing up a criterion we can make it more general by stating that the inverse should also be true: if the resource pot to a group of authorities **shrinks**, none of them should be **better off**.

Criterion:

- If the total amount of resource to be allocated to a group of authorities is changed (increased or decreased) then none of the resultant allocations to the authorities that are members of that group should change in the opposite direction.

Test:

- Remove each authority in turn, re-run the allocation mechanism, and check that in each case, all remaining authorities receive at least the same allocation as they would have received under the case with all authorities being present in the calculation.

4 Impact of authority reorganisations

In considering the impact of removing an authority, we also identified by extension that it would be useful to analyse the impact of merging authorities; this represents an alternative way of adjusting the levels in particular areas that is more reflective of likely real reorganisations of local government than an authority simply disappearing. (For example, it is expected that there will be a reorganisation of the authorities of Dorset in 2019 (Javid 2018).) In practice, for the current formula and associated datasets, constraints arise in merging the data from two or more authorities because these are not all simple additions. Several of the data variables included are expressed as proportions, such as the proportion of school children from particular ethnic groups. Calculating these proportion figures for any notional merger would require the values to be weighted based on the underlying variables in the denominators (in this example, the number of school children). The separate numerator and denominator variables that are used to calculate these proportion values are not typically provided in the dataset, so calculating the merged values is not possible.

Whilst the data availability constraints prevented an empirical this analysis of this issue in this case, from the understanding of the threshold mechanism by which the Wokingham effect occurs we can be confident that, at least in that case, if Wokingham were to merge with one or more neighbouring authorities, there would be a comparable impact on the formula's outputs; the resultant authority would not be the threshold authority, or at least would provide different threshold values, with the associated potential for large swings in unrelated authorities.

Criterion:

- In the case of merger the total allocation of the merged authority should equal the sum of those of the predecessor authorities. All other authorities' allocations should remain unchanged.

It could be argued that there may be a case for relaxing this criterion if there were felt to be particular economies (or diseconomies) of scale or scope that were intentionally to be reflected in the allocation mechanism. However, any such recognition would require particularly careful consideration and design, so as to avoid introducing perverse incentives.

Tests:

- Merge combinations of authorities and calculate resultant funding allocations.[4]

Recommendation:

- Data should be provided in a more raw format. Rather than providing values that exist as proportions, the numerators and denominators should be provided separately, with any proportions calculated as part of the calculation process, rather than treated as a data input. This would improve transparency in general, as well as facilitating tests like this.

5 Sensitivity of allocations to data updating

The recent mechanisms used to calculate funding allocations for local government in England have tended to be stable over a period of several years. In the most recent case, since 2013/14 and the introduction of 50 per cent business rates retention the allocations have continued to be made for several years largely without any updates to either the underlying formulas or to the data that were used in the formula. Consequently, it is important to understand the extent to which updating the data (whilst retaining the same formula) results in different allocations. The extent to which continued reliance on out-of-date data results in deviations from the allocations that would be found with updated data is an indication of the impact of not updating the data. To the extent that the formulas are professed to produce allocations that are in some sense the best available estimates of funding needs for each authority, then those impacts represent deviations between the funding that the authorities are receiving under outdated data, and what they 'ought' to be receiving under current data.

We have conducted various analyses related to updating the data. Due to data availability constraints, the updating has only been partial — only a relatively small proportion of the variables included in the 2013/14 funding formula system have been updated in our analysis. Table 3 lists the variables that have been updated. When we re-run the formulas using updated data we find (sometimes large) divergences from the funding allocations that are obtained using the original data. Throughout these analyses we consider the pre-damping allocations, to reflect the core assessment resulting from the formula, excluding the effect of the damping mechanism, which acts to suppress changes relative to pre-2013 allocations.

It is worth noting that the primary analyses presented here only include updates of data on the 'needs' side of the overall calculation, and do not include updates to the 'resources' element (broadly speaking, the council tax base for each authority), since an exact match for the 2013 formula's "Taxbase" variable is not

[4] In the English local authority context, examples of plausible combinations for testing might include pairs of shire districts within the same county; pairs of unitary authorities within the same ceremonial county; pairs of nearby London boroughs; pairs of nearby metropolitan districts; pairs of counties within two-tier areas; all shire districts and the county in two tier areas to create unitary counties.

Table 3: Variables updated. Short codes and full descriptive names.

Variable code	Variable long name
PROJALL	Projected Population in 2013
PROJ0TO17_12	Projected Population Aged 0 to 17
PROJ13TO19_12	Projected Population Aged 13 to 19 in 2010
PROJ18TO64_12	Projected Population Aged 18 to 64 in 2011
PROJ65PLHHSR12	Projected Household and Supported Residents Aged 65 and Over in 2011
MIDPOP	Resident Population at 30 June 2011
OTHERETH	People in Other Ethnic Groups
MIXETH	People in Mixed Ethnic Groups
KIDBLACKETH	Children in Black Ethnic Groups
OASPAR	Population Sparsity
LNOASPAR	Log of Population Sparsity
OPRENTED	Older People in Rented Accomodation
NOFAMILY	Households with No Family
SINGLEPAR	Single Parent Households
ETHSEC.NEW	Secondary Low Achieving Ethnic Groups
PUP3TO18_J10	Pupils aged 3 to 18
RESPUPALL	Resident Pupils aged 3 to 18

available in current datasets. Whilst every variable that is not updated in this analysis would, inevitably, result in some change of outcomes if it had been possible to update them, the omission of this variable merits particular attention since it might be expected to act in the opposite direction to the other variables: where population increases have been matched with increases in the number of households paying council tax, updating the resources component of the formula might act to (partially) offset some population-driven increases in assessed need. Due to the potential for this omission to create a systematic bias to the outcomes, this issue is investigated in an appendix with updated versions of the taxbase dataset that do not exactly match the definition from the 2013 formula. The results presented in the appendix show modest associations between the effect of updating the taxbase variable and the effect of updating all other variables, and that the direction of these associations is sensitive to which version of the updated taxbase data is used. This provides some comfort that the non-updating of this variable does not create an overwhelming systematic bias that would

Figure 1 presents the distribution of divergences that result from updating the listed variables. It should be noted that since these differences derive from only a relatively small subset of the variables being updated, the exact pattern of differences is unlikely to exactly match the pattern that would be obtained from a full updating of all variables. However, the exercise is useful in demonstrating the sorts of scales of divergence that are possible in a relatively small number of years. The largest increase is 27.1%; the largest decrease is -18.7%. These effectively represent authorities whose relative needs (as assessed by the formula) have shifted the most significantly, subject to the caveats about the assessment being based on a partial updating.

To examine the factors that influence the changed assessments under this partial updating, figure 2 shows how changes in allocations are associated with changes in population. It displays a general association whereby authorities that have experienced larger-than-average population growth tend to be identified for funding allocation increases, and those with lower-than-average population growth being associated with decreases in their assessed allocations, but these associations are not absolute, with many authorities varying from the general trend.

We undertook a form of 'hold-one-back' analysis to further understand the pattern of responses to updating variables that were displayed in authorities' allocations. Under this analysis we calculate the allocations that are obtained if we update all of the datasets that we have acquired updated versions of; we then also calculate the allocations that are obtained when we update all of the datasets except one. The differences between these notional allocations for each authority can be treated as an indicator of the influence of updating the

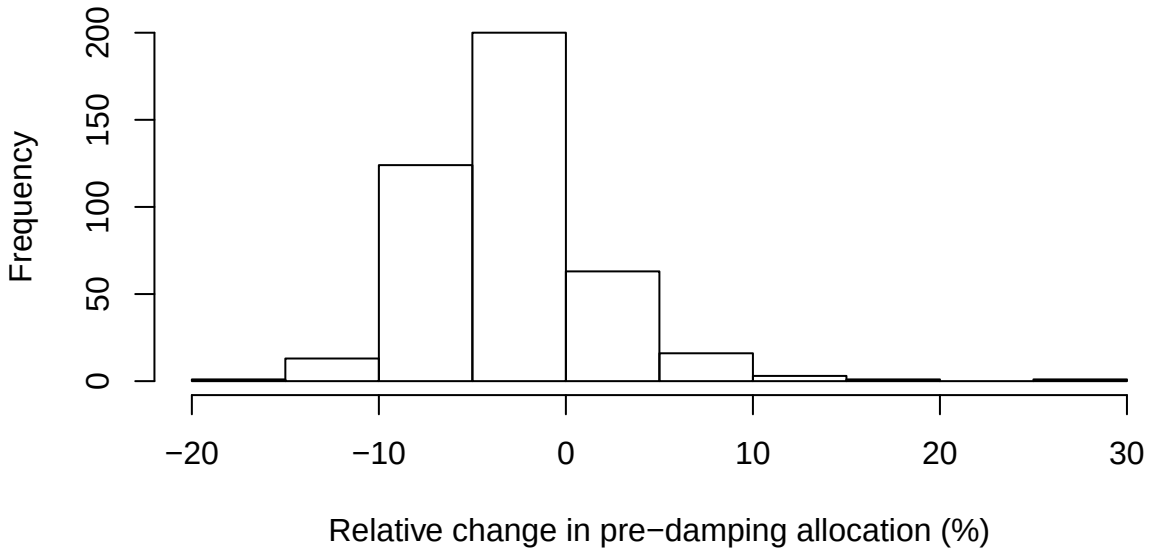

Figure 1: Distribution of pre-damping allocation changes from updating the tested variables

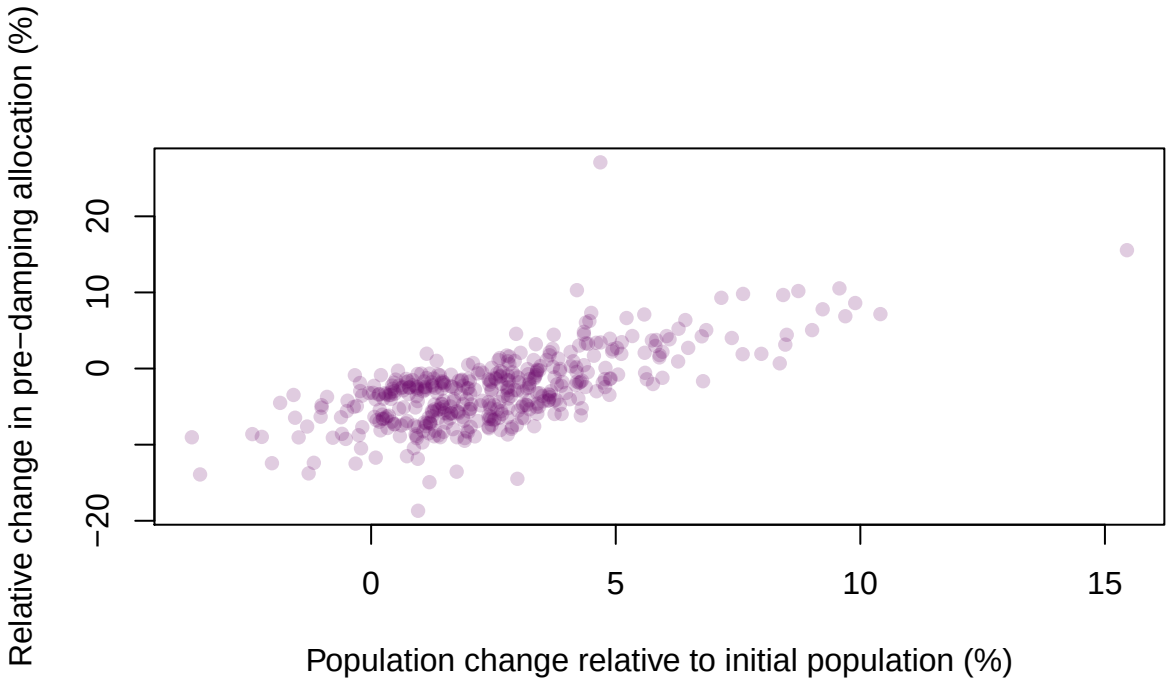

Figure 2: Pre-damping allocation changes from updating the tested variables vs. population changes

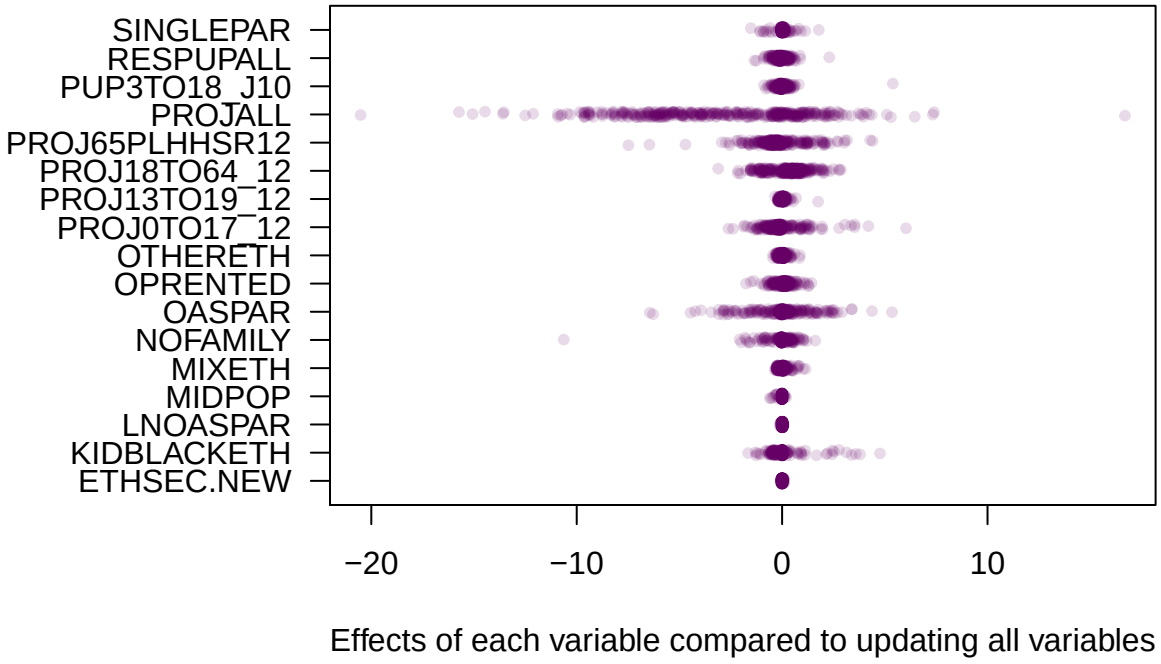

Figure 3: Effect of updates of each variable on each authority's assessed funding allocation from hold-one-back analysis.

held-back variable for that authority. For example, if we find that for a particular authority its funding formula increase would be, say, 5% if all variables were updated, but 6% if all variables except PROJALL were updated, then we can make an approximation that PROJALL is having a -1% effect for that authority. Figure 3 shows the effects of each variable by comparing what happens if we update them all to what happens if we update them all **except for the variable in question**. Although the PROJALL update is often a large factor in the overall swing for an authority, it is not always dominant. As can be seen from the figure, most variables result in at least some authorities experiencing substantially larger or smaller allocations.

As noted above, only a subset of the variables have been updated as part of this analysis; for all other variables the original data used in the 2013/14 exercise has been retained in the calculations. Given this constraint, and the fact that we find substantial deviations for at least some authorities when any given variable is not included, we cannot treat these findings as directly representing the results that would be obtained from a full updating. They are likely to be quite different from the results of either a full updating or of a different partial updating where a different subset of variables were accessed and updated. As the hold-one-back analysis shows, running these updates with any one of these datasets not updated would have resulted in at least somewhat different results; the same should therefore be assumed to be true of the variables that we have **not** been able to update: updating any one of these may have resulted in further notable differences in allocations. However, whilst this limits our ability to confidently determine how individual local authority allocations would have changed had the data been fully updated in recent years, we can draw important lessons about the process as a whole.

Firstly, the nature of the allocation mechanism's responses to partial updating provides reasons to be sceptical of findings from uprating the 2013/14 formula, even if it had been possible to update all variables. Whilst some of the variables updated have resulted in larger changes than others, few (or none) could be described as producing entirely negligible differences, and some make significant changes in some authorities but not others. The extreme sensitivity to various variables seems to indicate an unstable allocations mechanism that

may be more a product of an over-fit regression model than a robust assessment based on drivers of need.

Secondly, whilst it is hard to say whether a more reasonable formula would result in the same degree of sensitivity that we observe here, there are reasons to draw a broad lesson from these findings about the need for updates. Some of the input variables — including overall population levels that, *prima facie*, might be expected to be reasonably closely associated with shifting relative needs (especially if the demographics within those overall populations are relatively stable) — have experienced non-trivial changes since 2013/14. Crucially, this includes differing rates of change between authorities, making some shifts in funding need more than plausible.

Consequently the pattern of differences that arise from uprating based on the 2013/14 formula generates lessons both about the nature of that formula appearing to contain at least an element of arbitrariness, and about the need to not allow the data informing the assessment in any given year to fall too far out of sync with current knowledge.

Criteria:

- When running the funding allocation mechanism with data from different years, the results should not vary disproportionately to the underlying changes that are reflected in the data.
- Data should be updated with a frequency that reflects the tendency for assessed allocations to vary over time.

Tests:

- Test time series data backwards (i.e. use a few previous years' data in the formula) for excessive sensitivity / instability. Assess these for excessive and unexplained variation. A threshold could be set at, say, any shifts that are more than 1.5 times the magnitude of the **relative** population change (for example, if an authority's population has grown by 2% more than average, increases in its share of up to 3% would not require explanation).
- When adopting a new funding mechanism, it should be run with several years of historical data and the degree of deviation between authorities over the period analysed. The findings from this analysis should be published openly and the updating approach for future years should be set based on an understanding of what the implied rate of divergence in levels of allocations would have been, had that mechanism / formula been in place in previous years.

This should also have the benefit of encouraging good data and analysis behaviours, such as building the allocation tools such that updating (or backdating in this case) can be undertaken.

5.1 Data availability

In attempting to update the data to re-run the formula process, we sought to identify newer versions of the data that had been used in the original exercise. This uncovered the fact that many of the datasets used in the original exercise are not easily updateable. Issues include:

- Underlying context having changed (for example the change in the social security system to Universal Credit meaning that earlier definitions of unemployment-related data do not directly map onto current practice);
- Data published in formats that do not permit direct mapping to formula definitions (for example changes in the age categories that the data are published in being changed in a way that does not permit aggregation of the new data to match the versions in the formula);
- Use of commercial datasets where the providing organisation has changed its approach / classification methodology.

Criterion of continued availability of data:

- Funding mechanisms that are intended to be used for multiple years should be based on data that will continue to be published reliably in the future or where current estimates are expected to remain relevant for several years.

An example of a variable that **might** remain valid for an extended period could be data related to the average climate in an authority area, such as the average number of snow days, although even these may need to be reassessed over time, at a frequency that reflects the expected rate of climate change.

Test:

- Analysis indicating that the data series will continue to be available or that there is reason to believe that any point-in-time estimates will have sufficient longer-term validity as to be acceptable for inclusion.
- Each variable to be accompanied by documentation / meta-data indicting the publishing frequency of updates and an explanation of whether it will be acceptable to continue to rely on the data for the period implied by the future publishing schedule.

6 Duplicate indicators

Examining the formulas used in 2013/14 we identified areas where multiple variables appear to have been used as indicators of the same underlying issues. The following areas each have multiple variables that were used that seemed to be very close proxies for each other:

- **Population**: formula included both population projections and midyear population estimates (PROJALL and MIDPOP).
- **Sparsity**: variables calculating sparsity using similar methods but based on either the output areas or wards of each local authority (OASPAR and SPARSITY.NEW). The definitions of these also vary subtly based on the type of authority that the value is being assessed for, such that the value can be different for a county's police and fire authorities, even where these two organisations share the same boundaries. There is also a variable for population density, which may in part be quantifying some of the same features of authorities (in inverse).

It seems unlikely that there is a strong justification for including two slightly different indicators of population. Except where such justifications can be convincingly provided, there should be an assumption of parsimony in data that is meant to assess the same thing.[5]

Criterion of parsimony within constructs:

- For a given underlying construct that is important to include in the model, the single best dataset estimating that construct should be used consistently throughout.

Test:

- Calculate a matrix of correlations between each of the variables that are proposed to be used in the dataset. Assess any pairs of variables that have high correlation coefficients to examine whether they are really measuring different constructs. Explicitly document the justification for retaining any correlated pairs that are used in the final dataset.

Note: not all pairs of variables that have high correlations will inherently be measuring the same construct; many things — including the number of people in many categorical subpopulations (e.g. age cohorts) — will tend to increase at least somewhat in proportion to the overall population of the authority, for example. If particular sub-elements of a calculation are coherently and logically linked to the needs of given sub-populations, there may be a case for using data on the size of those sub-populations. Documenting justifications may act to clarify thinking about whether retaining closely-related pairs of variables is appropriate, and to make it transparent to subsequent users why the decision was made.

[5]There is also a question of whether parsimony of indicators and simplicity in general are desirable features of a funding formula. We do not address that question here.

7 Errors and inconsistencies

As part of our process of recreating the formula funding arrangements, we sought to rebuild the entire set of spreadsheets published by the government representing the 2013/14 formula funding process. Our intention in this was to test our new version of the model: being able to accurately reproduce the sheets in their entirety would act as a more robust test than, say, just testing the final output numbers. However, in conducting this test, as well as in other elements of our process of rebuilding the model, we identified various errors in the process as published.

Whilst these errors did not alter the eventual funding outcomes, they demonstrate the relative fragility of the system. Detailed descriptions of the errors identified are provided in the appendix, but briefly they included:

- Incorrect data, misstating the value for one authority by a factor of 1000; as the data value was not actually used in the calculations, it did not alter the allocations.
- Incorrect data, for another authority, using both the wrong underlying value **and** being out by a factor of 1000; the mistaken value was used to identify the band in which the authority fell, and it happened that the value ended up in the same band as the correct one would have, meaning that the ultimate allocations ended up unaltered.
- An incorrect column of calculations, meaning every authority of the class appeared to have the value that should have been associated with the next authority down the list, and one authority of a different class having a value where it should not have had one.
- Data or calculations in cells for various categories of authorities where these values are irrelevant (multiple occurrences).
- Missing data, for individual cells or for all authorities of a class within a column.
- Calculated totals for some classes of authorities in some columns calculated incorrectly.

We also identified other problems that, whilst falling short of being actual errors, do not reflect best practice:

- Summary totals for fixed value columns calculated inconsistently as either the sum of all occurrences of the (same) value, or as just the value itself (i.e., the difference between a column containing 400 values all of 10 would have the total of 10 or 4,000).
- Sheets that mix data and calculated values.
- Inconsistency in variable labelling between sheets.

These tools are developed, maintained and operated by humans. Given their complexity, the introduction of errors or inconsistencies is almost inevitable. In developing our own version of the model there were certainly instances where the intended output was not produced first time. Quality control approaches are needed to minimise these, and should be combined with approaches that lower (though not entirely eliminate) the likelihood of mistakes in the first place.

Criterion:

- Adoption of practices that minimise chances of error and/or maximise chances of errors being detected prior to final deployment.

Tests:

- Clean-room parallel implementation of the system from the specification (possibly using different tools for each). Check that the same results are produced by each.
- Data should be tested for being within plausible ranges. This process should ideally include automated numerical techniques and may also include visual techniques (examining plots of data to visually see potential outliers. Any outliers identified should be examined to detect whether the data represents a true outlying value or an error requiring correction. Many variables will fit into one of two categories:
  - Variables where plausible values fall within a window and should be at least relatively similar across authorities (e.g., where the variable represents some proportion of people such that the numerator is a subset of the denominator, the value must be between 0 and 1, and the plausible range might be even smaller);

– Variables that increase loosely in line with population of the area, so by scaling the variable for population the values for all authorities should fit within a plausible window, even if the unscaled versions vary by factors of 50 or more due to the range of populations of areas.

8 Non-transitions

Having identified (an intended indication of) the differing levels of funding need between authorities, one of the final stages of the 2013/14 funding formula process was 'damping' — applying factors that prevented authorities from losing out to more than a particular extent (dependent upon their class of authority and their level in previous years). This process appears designed to ensure that no authorities received (proportionately) excessive income shocks in 2013/14, whilst moving them all a degree closer to the 'correct' allocation as indicated by the formula. However, the logic of this process has not been followed through in subsequent years. Further funding has effectively been on the basis of starting from the post-damping 2013/14 allocations, and reducing all authorities of a class proportionately; those that were, pre-damping, closer to their intended target allocation received the same levels of cuts as those that were assessed to be further from it.

Criterion:

- Where funding allocation mechanisms are designed such that they feature deviation between assessed allocations and actual allocations on the grounds of providing protection against income shocks, the mechanism should also be designed such that those deviations would reduce over time.

Test:

- Analysis of the design of the mechanism should show that the process by which any transitional arrangements will be un-wound over time is clearly detailed.

9 Conclusions and recommendations for practice

This paper has described the findings from an examination of the 2013/14 funding allocation mechanism for local government in England (which remains in force several years later). As described above, we were able to rebuild the funding allocation model in a different system (the R language), validate our version by checking its results against the original, and deploy it with variations on the data input in order to test various features of the model. These tests included re-running the funding formulas with updated data, where this was available. Because of the sensitivity demonstrated by the updating we have been able to conduct, we are not able to say confidently that the outputs produced by these updatings accurately reflect the magnitudes — or even the signs (+/−) — of the changes that any **individual** authority would have received if all of the data in the formula were updated. The findings should not, therefore, be relied upon to indicate that any given authority is a 'winner' or 'loser' from the data not having been updated. They do, however, provide a good indication of the degree of sensitivity that the formula process has to the updating of data.

We were also able to replicate an effect — "the Wokingham Effect" — that had been known for several years, and will have had an influence on local government funding in England for more than a decade. The story of this effect is particularly stark. It was identified in 2008 and had been described in a policy-focused publication by 2009 (Stone 2009). It was described in detail in a journal article in 2011. And yet, it persisted into the formula that was in place for 2013/14, and by extension will have remained within the basis of local government funding in England until 2020. Despite producing results that are incongruous, the underlying funding mechanism retains a design that can generate this output. The capacity required to update a funding allocation mechanism for a country like England is significant, and there are features of stable systems that are desirable, so it is entirely understandable that features of these mechanisms are relatively persistent. This makes it essential that when changes are made to the mechanism, the lessons from previous exercises are

learnt and formally included in the process in the form of criteria and tests that any new variant mechanisms can be checked against.

The government is currently in the process of designing the formula funding system that will establish the resources available to the various authorities that make up English local government from 2020/21 onwards. The issues that have been highlighted by our review of the system in place from 2013/14 provide clear indications of measures that should be adopted to avoid similar problems being repeated. Table 4 summarises the criteria that we have derived from our examination of the existing funding mechanism and the tests that should be employed when devising any new mechanism.

Table 4: Summary of problems, criteria and tests.

Problem	Criterion	Test
Adding resources can result in losses (aka "The Wokingham Effect")	If the total amount of resource to be allocated to a group of authorities is changed (increased or decreased) then none of the resultant allocations to the authorities that are members of that group should change in the opposite direction.	Remove each authority in turn, comparing allocations to the case with all authorities.*
Impact of authority reorganisations	In the case of merger the total allocation of the merged authority should equal the sum of those of the predecessor authorities. All other authorities' allocations should remain unchanged.	Merge combinations of authorities and calculate resultant funding allocations.*
Sensitivity of allocations to data updating	Results should not vary disproportionately to underlying changes reflected in data from different years	Test timeseries with historic data, assessing for excessive sensitivity/instability and unexplained variation.*
	Data should be updated with a frequency that reflects the tendency for assessed allocations to vary over time	Use historical data to identify the degree of deviation over the period analysed and inform design of updating process / frequency.*
Data availability	Funding mechanisms should be based on data that will continue to be published reliably in the future	Analysis indicating that the data series will continue to be available or that any point-in-time estimates will have sufficient longer-term validity.†
		Documentation for each variable indicting the publishing frequency of updates and explanation of acceptability of relying on the data according to that publishing schedule.†
Duplicate indicators	The single best dataset estimating each underlying construct should be used consistently throughout	Calculate correlations between each of the variables that are proposed to be used in the dataset. Assess any high correlation coefficients. Document the justification for any correlated pairs that are retained in the final dataset.*

Table 4: Summary of problems, criteria and tests. *(continued)*

Problem	Criterion	Test
Errors and inconsistencies	Adoption of practices that minimise chances of error and/or maximise chances of errors being detected prior to final deployment	Clean-room parallel implementation of the system from the specification, checking results match.[*] †
		Testing data for being within plausible ranges, including automated numerical techniques and possibly visual techniques.[*]
Non-transitions	Where a mechanism is designed to feature deviation from assessed allocations to provide protection against income shocks, those deviations should reduce over time	Design of the mechanism should clearly detail the process by which any transitional arrangements will be un-wound over time.†
Key:		
[*] Data-driven tests.		
† Design tests, typically requiring manual analysis of process, variables, etc.		

It would be particularly powerful if these criteria and tests were adopted early in the process of designing a mechanism, along with a commitment to document and publish the findings of the tests. This level of transparency would contribute significant credibility to the process. This would complement transparency from other parts of the process, such as providing the datasets in a relatively raw forms. As detailed above, if numerators and denominators of any 'proportion' indicators were provided separately, and the proportions calculated as part of the calculation process, this would improve transparency in general, as well as making certain types of tests easier to conduct.

In drawing up the criteria we have taken efforts to make them general rather than specific. They aim to be 'mechanism neutral', ruling out only consequences that are clearly undesirable or illogical, rather than directly prohibiting any given class of allocation mechanism that might be used. Some of the identified problems — such as the "Wokingham effect" — occur via mechanisms that are very well understood and documented (in that case, due to the use of thresholds in the model). We aim to avoid solutions that relate too specifically to those erroneous mechanisms, and instead refer to the preferred outcomes that we believe are more defensible. In the case of the effect of thresholds on the model, adopting a criterion that said "no thresholds" is less general than one that prohibits the class of behaviour that can be exhibited when thresholds are invoked. Avoiding the inappropriate use of thresholds might be *necessary* to avoid these types of problems, but would not be *sufficient*, since other mechanisms could also trigger the same class of allocative misbehaviour.

The allocation of substantial public resources rests on the implementation of a suitable mechanism. The adoption of criteria and tests such as the ones described in this paper provides a valuable way of assessing whether any given mechanism can be considered suitable. Future work could also usefully extend the range of criteria and tests available, to build an even more comprehensive battery of assessments in support of a justifiable division of resources.

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Appendices

A Errors and inconsistencies

This appendix provides more detail on the errors and inconsistencies identified in the spreadsheet versions of the 2013 funding formula mechanism. The funding formula spreadsheet was published by the Department for Communities and Local Government, and made available for download via its 'Local Government Finance Settlement 2013/14' webpage; the webpage is now archived at http://webarchive.nationalarchives.gov.uk/20140505104701/http://www.local.communities.gov.uk/finance/1314/settle.htm and the Formula Funding model Excel download is archived at http://webarchive.nationalarchives.gov.uk/20140505104701/http://www.local.communities.gov.uk/finance/1314/1314FFmodels.xls.

The Relative Needs Formula (RNF) spreadsheet does not appear to have been formally published by the Department for Communities and Local Government. It was shared with selected organisations identified by government as 'key players' in early 2013, and was intended to enable users to undertake some forms of "what-if?" analysis. The Notes to the RNF Excel workbook explicitly stated that the Local Government Finance Report was the definitive source if the spreadsheet differed from it.

A.1 Incorrect data

Workbook	Error location	Notes
FF	`Damping!F385`	Correct value believed to be 437300000 (see below). Consequent errors in cell `H385` and several rankings in range `I355:J385`, plus totals in `F426`, `F431`, `F453` and `F456`.
FF	`Damping!F423`	Correct value believed to be 2673300000 (see below). Consequent errors in totals in `F426`, `F431`, `F452` and `F456`.
FF	`Data!AI523`	Correct value believed to be 248.267204 (see below). Consequent error in total in `Data!AI563`.

The 'Damping' sheet of the Funding Formula Excel workbook includes some data fields relating to previous years' budgets and grants. Columns F and G are titled "2010-11 Budget Requirement" and "2010-11 Formula Grant", respectively. Column H calculates "2010-11 Formula Grant as proportion of 2010-11 Budget Requirement" (column G divided by column F). Most of the resultant (proportion) values fall in the range between 0.187 and 0.876, indicating a reasonable degree of variation, but all within the same order of magnitude (i.e., just under 20% to just under 90%). However, for two authorities — 'GLA - fire' and 'GLA - police' — the proportion values are three orders of magnitude higher — `440.180` and `758.515`, respectively (cells H385 and H423).

These anomalous results result from incorrect values for these authorities' 2010-11 Budget Requirements: cells F385 and F423 contain `589700` and `2673300` respectively. Unfortunately, there does not appear to be a source specified for the "2010-11 Budget Requirement" field in the formula definition or data definition documents, so to find the correct values we referred to the original budget document of the GLA from 2010-2011 (Greater London Authority 2010), and since these contained "proposed" values these were double-checked against the 2011-2012 budget document (Greater London Authority 2011) to confirm that the proposed values were accepted. The value in the spreadsheet for the 'GLA - police' authority seems to be incorrect to exactly a factor of ×1000 (as if the value had been entered as £thousands instead of as £): the Metropolitan Police Authority budget requirement is £2,673.3m (Greater London Authority 2010, p14). The value for the fire authority, however, appears to be incorrect in two ways. As well as being incorrect by a factor of ×1000, it also seems that the wrong base number features in the spreadsheet: the budget requirement of the London

Fire and Emergency Planning Authority is given as £437.3m (Greater London Authority 2010, p19). It appears that the most likely source of the £589.7m value used for the fire authority is the total budget requirement for all components of the GLA except for the Metropolitan Police Authority, which is provided in a budget requirement summary table (Greater London Authority 2010, p41).

Due to the banding system used for calculating the damping mechanism, which was in force for the fire authorities but not police authorities, incorrect data in relation to the fire authority had particular potential to influence the eventual allocations: had the error been such that it had moved the GLA fire authority between bands it would have had a knock-on effect on most of the fire authorities. For example, if the spreadsheet had been set up with the incorrect £589.7m value, but without the incorrect factor of 1000, the allocations shown in the table 6 would have resulted.

Table 6: Example of difference in allocations that can result from incorrect data in fire authorities' damping mechanism.

Authority code	Authority name	Example allocation (£million)	Actual allocation (£million)	Difference (£)
R301	Greater Manchester Fire	61.203390	61.158642	44748
R302	Merseyside Fire	36.959178	36.959178	0
R303	South Yorkshire Fire	29.349180	29.349180	0
R304	Tyne and Wear Fire	28.390669	28.390669	0
R305	West Midlands Fire	64.644797	64.637863	6934
R306	West Yorkshire Fire	47.424215	47.404716	19499
R950	Avon Fire Authority	20.447768	20.447768	0
R951	Cleveland Fire Authority	17.864439	17.864439	0
R952	Humberside Fire Authority	23.667397	23.650846	16551
R953	North Yorkshire Fire Authority	11.274836	11.254500	20336
R954	Bedfordshire Fire Authority	10.436710	10.401054	35656
R955	Buckinghamshire Fire Authority	9.234627	9.234627	0
R956	Derbyshire Fire Authority	16.084269	16.084269	0
R957	Dorset Fire Authority	10.069436	10.056027	13409
R958	Durham Fire Authority	11.916735	11.898415	18320
R959	East Sussex Fire Authority	12.998131	12.962177	35954
R960	Hampshire Fire Authority	27.020504	27.020504	0
R961	Leicestershire Fire Authority	17.232707	17.123943	108764
R962	Staffordshire Fire Authority	17.241551	17.228411	13140
R963	Wiltshire Fire Authority	8.456468	8.441580	14888
R964	Berkshire Fire Authority	13.435318	13.435318	0
R965	Cambridgeshire Fire Authority	11.338961	11.338961	0
R966	Cheshire Fire Authority	17.324311	17.299273	25038
R751	Devon and Somerset Fire Authority	28.128534	28.112595	15939
R968	Essex Fire Authority	29.726883	29.726883	0
R969	Hereford and Worcester Fire Authority	9.662038	9.646653	15385
R970	Kent Fire Authority	26.127940	26.090690	37250
R971	Lancashire Fire Authority	28.878395	28.842438	35957
R972	Nottinghamshire Fire Authority	19.634066	19.634066	0
R973	Shropshire Fire Authority	6.448184	6.444966	3218
R572	GLA - fire	241.572698	242.053683	-480985

The Data worksheet of the FF workbook includes a value of 447.266952630294 in the "2013-14 Police Grant" column for Greater Manchester (AI563). This error was found because there is a near-duplicate column containing the same data in the Pre-damping worksheet; the alternative version contained the same data for all other authorities, but a different value (248.267204) for Greater Manchester. It was possible to confirm which value was correct by reference to the calculation documentation (Department for Communities and Local Government 2013a, p42, paragraph 3.20(c)), which provided the reference for the ultimate source of the correct values (Home Office 2013, column (a) of table in paragraph 3.1). Because the formulas in the subsequent stages used the correct version of the data, the ultimate allocations were unaffected; had the incorrect version been referenced it would have resulted in differences in many of the police authorities' allocations.

A.2 Incorrect column of calculations

Workbook	Error location	Notes
RNF	Summary!AI327	Contains value derived from data in row below (row 328). Should be empty.
RNF	Summary!AI328:AI451	Each cell contains value derived from data in the row below.

The Summary worksheet of the RNF workbook contains an incorrect column (AI) summarising the calculations of the component of the RNF that relates to the Upper-tier. The calculations each pick up the sum of the constituent values for the next row down. Values should only be present for upper-tier authorities (or the upper tier row for authorities with both lower- and upper-tier responsibilities); due to the upward shift in calculations, the first entry (which contains the value for the first upper-tier authority) is on the row for the **last** lower tier authority.

A.3 Irrelevant data included

Workbook	Error location	Notes
FF	Pre-damping!O2:O477	Contains duplicate of Data!E2:E477.
FF	Pre-damping!AP2:AP561	Contains (near) duplicate of Data!F2:F561.
FF	Data!AF327:AF450	Contain 0 values. Should be blank.
FF	Pre-damping!BP523:BP561	Contains (near) duplicate of Data!AI523:AI561.
RNF	Fire!D397:H397	Contain duplicates of Fire!D520:H522.
	Fire!D443:H443	
	Fire!D445:H445	
RNF	Fire!D468:H478	Contains duplicate of Fire!D509:H519.

The Pre-damping sheet of the Formula Funding workbook contains a column (O) that partially duplicates the "2013 Population Projection" column (E) of the Data worksheet by means of a formula referencing the named range for the Data version (=Data!midpop). Similarly, it contains a column (AP) that almost entirely duplicates the "taxbase" column from the Data worksheet (F), again by a named range reference (=taxbase). In most cases the 'calculations' worksheets simply reference the data that they are using in their formulas, without duplicating it. In fact, the subsequent column in the Pre-damping sheet that uses the population and taxbase data (column AQ) does not refer to the duplicated versions on the Pre-damping worksheet; it still uses the named ranges in its formula (=taxbase/midpop) to refer to the main versions in the Data worksheet. The same applies to the various "per head" columns (P, T, X, AB, AF, AJ) and columns BI and BN, all of which reference the midpop named range to directly access the population projection data from the Data worksheet.

The Pre-damping worksheet does include more totals rows than the Data worksheet, but since these are not correctly calculated (see below) this does not seem to provide a justification for including the duplicate column.

The Data worksheet of the FF workbook contains 0 entries for the upper-tier versions of authorities with both lower- and upper-tier responsibilities (AF327:AF450). In fact, each of these authorities does have a non-zero value for the mixed-tier RNF, as can be seen in their lower-tier entries (AF203:AF326); including it in both the lower- and upper-tier entries would result in double-counting, so it makes sense to notionally apportion it to one tier. It would be better to achieve this by not providing any values (i.e., having empty cells), since the zero entries are slightly misleading. Blank cells would also be more consistent with other handlings on the same sheet, including the two columns (Y and Z) that are added to calculate this mixed-tier RNF.

As described above, an error was discovered in the version of the Police Grant data on the Data worksheet of the FF workbook due to there being a near-duplicate version of the data in the Pre-damping worksheet.

The Fire worksheet of the RNF workbook contains duplicate entries listed alongside the irrelevant instance of several authorities. For the three unitary authorities with fire responsibilities (those listed as class "UNIFIR" in column C), part of the fire data is reproduced alongside their upper-tier instances (rows 387, 443 and 445). For the eleven shire counties with fire responsibilities (class "SCFIR"), the data are duplicated alongside their non-fire instances (rows 468 to 478).

A.4 Missing data

Workbook	Error location	Notes
FF	Pre-damping!O478:O561	Missing data for police and fire authorities. Should contain named range reference (=Data!midpop). Consequent errors in totals in AP577, AP583, AP591 and AP593.
FF	Pre-damping!AP198	Missing reference to data source. Should contain named range reference (=taxbase). Consequent errors in totals in O568, O569, O574, O575, O584, O585, O589, and O590.
RNF	Pre-damping!O478:O561	Missing data for police and fire authorities. Should contain named range

As described above, there are seemingly-unnecessary copies of the population projection and taxbase data in the Pre-damping worksheet. However, these copies are also incomplete. The entries in the population column are missing for the police and fire authorities (O478:O561) and the cell in the taxbase column that relates to Harrogate (AP198) is empty. Since the calculations on the worksheet refer to the original data not these columns, the missing values have no impact on the outputs. Some of the column totals for the columns are also affected by the missing values (though see below since these totals are mis-calculated anyway).

As described above, there is an error in the calculation of the "Upper-tier RNF" component (column AI of the Summary worksheet in the RNF workbook). The entries in the column are also only provided for those authorities that have both lower- and upper-tier responsibilities. Although the reason for including the column is not explicitly provided, it appears at least arguable that the column should have been expanded (once fixed) to also provide values for the shire counties (i.e., the authorities with **only** upper-tier responsibilities). Furthermore, whilst columns are provided summarising the components of the RNF that relate to upper-tier and mixed-tier responsibilities (AI and AK, respectively), and the fire and police RNF components have their own columns (O and Q), there is not a column that specifically contains the lower-tier component for authorities with those responsibilities. If these tier / responsibility component summaries are valuable, it seems likely that a lower tier one would be as valuable as any of the others. The FF workbook does include a lower-tier RNF column (Data!AE), and does extend the upper-tier RNF column (Data!AB) to cover the shirt

counties as well as the upper-tier components of those authorities with multiple-tier responsibilities.

A.5 Incorrect calculation of totals

Workbook	Error location	Notes
FF	`Pre-damping!O563:O593` `Pre-damping!AP563:AP593`	Double counting of individual authorities' data and of data that is included in authorities covering the same area.

FF	`Pre-damping!P563:R563`
	`Pre-damping!T563:V563`
	`Pre-damping!x563:Z563`
	`Pre-damping!AB563:AD563`
	`Pre-damping!AB563:AD563`
	`Pre-damping!AK563:AM563`
	`Pre-damping!AQ563:BH563`
	`Pre-damping!BK563:BM563`
		London Area totals (TL) provided as 0 for several columns where other totals not provided. Should probably have been left blank (as for other totals).

RNF	`Summary` sheet, cells `Q584`, `Q586`, `U569`, `U570`, `U577`, `U586`, `X569`, `X570`, `X577`, `X586`, `Z577`, `Z586`, `AA569`, `AA570`, `AA577`, `AA586`, `AC569`, `AC570`, `AC577`, `AC586`, `AE569`, `AE570`, `AE577`, `AE586`, `AG569`, `AG570`, `AG577`, `AG586`, `AK569`, `AK570`, `AK577`, `AK586`.	Various totals calculated from wrong tier or tiers. Consequent errors in other totals that are derived from these ones.

RNF	`EPCS - CT!E584`
	`EPCS - CT!E586`
	`EPCS - CT!F584`
	`EPCS - CT!F586`
	`EPCS - Bits!Q584`
	`EPCS - Bits!Q586`
	`EPCS - Bits!R584`
	`EPCS - Bits!R586`
		Various totals calculated from wrong tier. Consequent errors in other totals that are derived from these ones.

Totals are included on the `Pre-damping` sheet for the "2013 Population Projection" and "Taxbase" data (columns `O` and `AP`, respectively). These are miscalculated, double-counting populations (and taxbases) in various ways.

The formulas for calculating the sums of the values for authorities of a given class, such as "ILB" (Inner London Boroughs) and OLB (Outer London Boroughs) identify values associated with all instances of that class. Where these authorities appear twice (for their lower- and upper-tier responsibilities), both versions are included in the calculation. Consequently the totals provided for authorities such as the London boroughs are double what they should be.

In addition, the population of an area is counted in all the authorities with responsibility for that area. Consequently, a person might be counted in the population figures for a shire district, a county council, a police authority, and a fire authority. The totals formulas act in such a way that they will multiply count these populations. Consequently, most of the totals values include (at least) double counting, and the total for England ends up several times larger than it should be.

The same double-counting applies to the taxbases of areas. More informative totals for the taxbases might have been obtained by providing values on the columns providing the taxbase shares to different tiers of authorities (columns `AR`, `AV`, `AZ` and `BD`), but the totals rows for these columns are blank.

For many of the columns on the Pre-damping sheet, most of the totals are not provided. The omission of these totals appears to be reasonable. However, the formula for the TL (total for the London Area) has been left in place, providing 0 values in each of these columns since it refers to the (omitted) totals for the types of authorities that comprise the overall London area. This is an error in the sense of having some totals entries where it appears that the intent was to not provide totals, and also in the sense that most of these values should not be 0 if totals were to be provided.

Many of the total values on the RNF workbook's Summary worksheet are calculated by reference to the wrong tier or tiers. Unitary authorities, metropolitan districts, London boroughs and the City of London have both lower- and upper-tier responsibilities so are represented within the sheet with a row for each of these tiers. Similarly, shire counties with fire responsibilities have rows for both their upper-tier responsibilities and in their roles as fire authorities, and unitaries with fire responsibilities have three rows. In some cases, the data relating to a given column is contained in the authorities' lower-tier or fire rows, and the total references the upper tier rows. In other cases the total formula should reference all of the relevant rows (multiple rows per authority) since different parts of the data are accounted for separately under the authorities' different responsibilities. A smaller number of similar errors exist in the EPCS - CT and EPCS - Bits worksheets.

A.6 Inconsistent calculation of totals

Summary totals for fixed value columns in the RNF workbook are calculated inconsistently as either the sum of all occurrences of the (same) value, or as just the value itself. For example, the TL (Total London Area) total for the column CEF Fixed Cost Amount (Children's Services!U567) is given as the fixed value, not the sum of these fixed values. Conversely, the other totals in that column (U569:U596) do provide sums of the fixed amounts. For the basic amounts columns the London totals are calculated as sums, as are the totals below; see: "Youth and Community Basic Amount" (Children's Services!G567), "Pupils Basic Amount" (Children's Services!P567), "Resident Pupils Basic Amount" (Children's Services!Q567), and "Children's Social Care Basic Amount" (Children's Services!AF567). Arguably, where a a constant value is going to be multiplied by authority-specific data, the more logical treatment is to provide the fixed value in the totals rows, or simply to refrain from providing totals in these cases. Whichever approach is adopted, it should be consistent.

A.7 Data and calculations mixed

Workbook	Error location	Notes
FF	Data!AB327:AB477 Data!AE2:AE326 Data!AF2:AF477 Data!AH2:AH561	The Data sheet contains columns of derived / calculated values, each a sum of relevant earlier columns.
FF	Pre-damping!E327:K477	Contains data (not calculated) values for various allocations and grants.
FF	Pre-damping!L522	Contains data duplicated from Data!P522
FF	Pre-damping!BP523:BP561	Contains (near) duplicate of Data!AI523:AI561.
FF	Damping!F2:F424 Damping!G2:G424 Damping!L2:L424 Damping!T203:T353	Contains data (not calculated) values related to previous years etc.

In general, the workbooks adopt a sensible practice of separating data from calculations. Each workbook contains a Data worksheet and some other sheets that include fixed values that can be treated as data, and most of the calculations on the remaining sheets draw on these sources for their calculations. However, the approach is not followed with complete consistency, and there are calculations included in the data and data included with the calculations.

Four of the columns that are the results of summing RNF components ("2013-14 Upper-tier RNF", "2013-14 Lower-Tier RNF", "2013-14 Mixed Tier EPCS RNF", and "2013-14 Total RNF") are presented in the FF workbook as calculations. This is a reasonable approach, as the constituent parts are included in the workbook. However, the calculations are included in the `Data` worksheet. Including these calculated variables in a worksheet that is explicitly labelled as containing data is potentially confusing and more likely to result in errors in updating, maintaining or using the spreadsheet.

Conversely, there are several instances of the non-data sheets containing data columns. The `Pre-damping` worksheet includes seven columns (`E327:K477`) that provide data on various allocations and grants related to upper-tier authorities ("Local Transport Services Allocations", "Supporting People Allocations", "Housing Strategy for OP", "LSC Staff Transfer", "AIDs grant allocation", "Preserved Rights Grant", and "Animal Health & Welfare Allocations"). The "County-Level Civil Contingency Functions in London" value (a single value of 0.5) is duplicated from `Data!P522` to `Pre-damping!L522`. The `Pre-damping` version is the one that is used in the subsequent calculations. The `Damping` worksheet contains four columns of data, including data on previous years' budgets and grants (`F2:F424`: "2010-11 Budget Requirement", `G2:G424`: "2010-11 Formula Grant", `L2:L424`: "Adjusted 2012/13 Formula Grant", and `T203:T353`: "Central Education Functions within LACSEG transfer").

As described above, an error was discovered in the version of the Police Grant data on the `Data` worksheet of the FF workbook due to there being a near-duplicate version of the data in the `Pre-damping` worksheet. Whilst the correct version of the data happened to be on the `Pre-damping` worksheet, the more logical location for the data would be the `Data` worksheet.

A.8 Inconsistency in variable labelling

Where the same variable is used it is not always consistently named. An example of this can be found in the FF workbook, where the same variable changes name from "Formula Funding before Floors" to "Formula Grant before Floors" when it gets aggregated to the authority level (`Pre-daming!BQ1` and `Damping!E1`).

B Updated taxbase calculations

The primary data updating analyses presented in this paper are focused on those datasets where newer versions are available that exactly match those used in the original formula. In many instances the closest available updated datasets are not direct matches, for reasons such as changed age boundaries or differences in underlying definitions, and hence have not been included in the updating analyses. Adopting an approach of only using exact matches has the advantage of making the subsequent inclusion decisions all be fairly objective — is a given updated dataset a direct match for the previous one? — rather than having to subjectively decide whether a near-match is 'close enough'.

The general lessons from the main updating analysis are that updating most variables will result in relatively large swings in allocations for at least some authorities (rather than being overly focused on any specific swing for any particular authority). Consequently, in most cases the constraint that exact data matching places on excluding some variable from updating is an acceptable trade-off. However, in the specific case of the taxbase variable there was, *prima facie*, a reason for thinking that its effects might be **systematically** different from the general trends of other variables. Increases or decreases in assessed relative need for authorities could, to some extent, be expected to reflect relatively larger or smaller growths (or declines) in authorities' populations (or relative subpopulations); those authorities experiencing the highest population growth may tend to have growing need as assessed by the formula across a range of areas. To the extent that there could be a systematic pattern for other variables, the most likely expectation in most cases would be that they would tend to also be (somewhat) associated with population changes in this way, and would hence tend to amplify the changes observed through the already-updated variables. For taxbase, however, there was reason to suspect that the relationship might be **opposite** in direction: higher population growth might be associated with higher taxbase growth, and a larger taxbase would tend to **decrease** the assessed funding

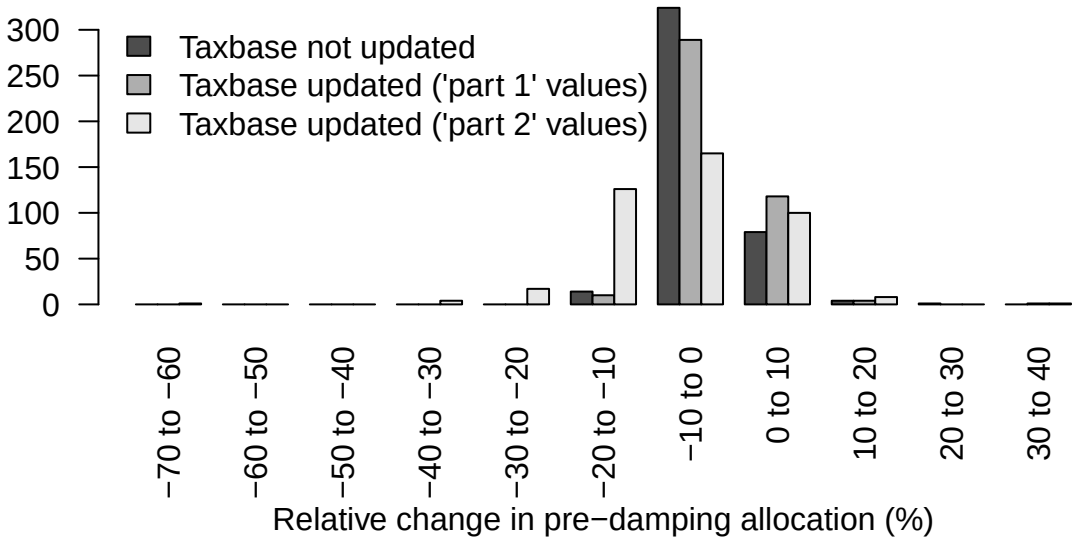

Figure 4: Comparison of distributions of pre-damping allocation changes from updating the tested variables, with or without taxbase updating.

requirement. This potential for a systematically opposite effect suggests that analysis of updated taxbase statistics would be important, even though the datatset is not an exact match.

The taxbase data used in the 2013 funding formula calculations is based on a specific definition that applies a number of adjustments to the raw taxbase values. This adjusted value is not presented in more recent taxbase datasets published by the government. There is further uncertainty because the underlying systems that these data represent has also changed; the most significant change has been in the operation of the council tax support system for low income households, which has moved from being a central government benefit to a system of localised support. The current taxbase dataset includes versions for taxbase both before and after accounting for this support; not only do neither directly match the version previously used in the formula, the conceptual change in council tax support means it is also not certain which version would be deemed most appropriate as the nearest substitute. Consequently, the analyses were re-run using both of the two main taxbase value presented in the current government datasets, the "Part 1" value (which is listed as "Tax base") and the "Part 2" value ("Tax base after allowance for council tax support"). Figures 4 shows the distributions of the changes in allocations, depending on whether there is no updating of the taxbase variable, or if each of the two options is used. The 'part 2' version shows a higher proportion of larger variations.

The results of the hold-one-back analyses using the two variants of taxbase data are shown in Figures 5 and 6. As with updating other variables, the taxbase updates result in a spread of results, varying between authorities, and contributing relatively large changes in some cases. More importantly for investigating whether updating the taxbase variable would result in a **systematic** difference to the pattern of outcomes, we can examine the relationship between the effects of updating all variables except taxbase, and the effect of the taxbase update. As can be seen in Figures 7 and 8, the associations between the effect of updating all variables and the effects of updating the taxbase variable are not overwhelming. Statistically, the R-squared measures for these associations are 0.160 and 0.158 (for taxbase part 1 and 2, respectively). Importantly, they also vary in direction, depending on which updated version of the taxbase variable is chosen, with the

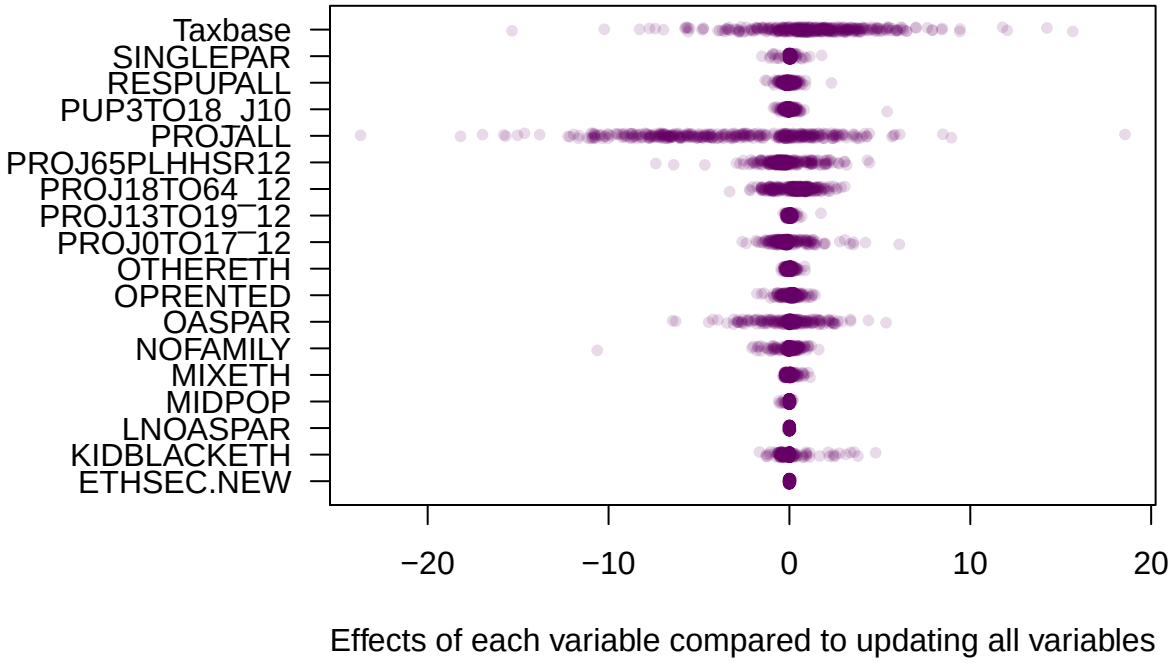

Figure 5: Effect of updates of each variable (including 'part 1' taxbase values) on each authority's assessed funding allocation from hold-one-back analysis.

respective gradients being -0.614 and 0.297. The modest strengths of the associations observed, and the variation dependent upon how the updated taxbase is specified, provides some comfort that not updating this variable is not introducing large systematic bias to the analysis of the effects of updating data. However, as with all of the datasets that have not been updated for this analysis, it is clear that there would be some further alterations to the specific values that would be obtained for each authority.

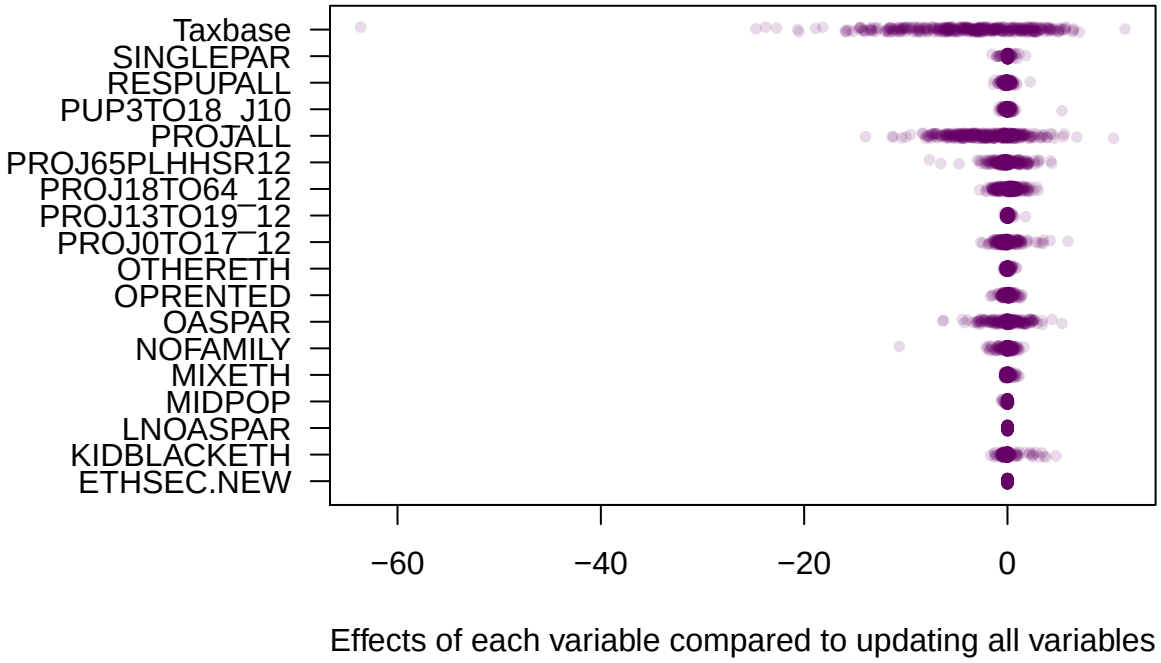

Figure 6: Effect of updates of each variable (including 'part 2' taxbase values) on each authority's assessed funding allocation from hold-one-back analysis.

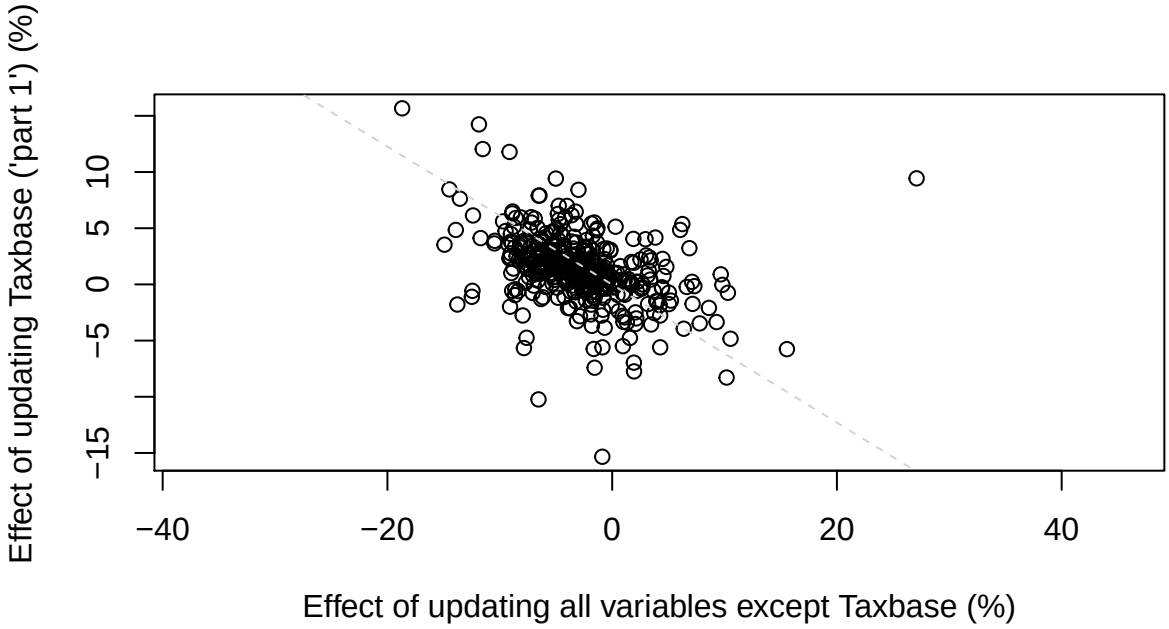

Figure 7: Effect of updating Taxbase with 'part 1' values compared to effects of updating all other variables.

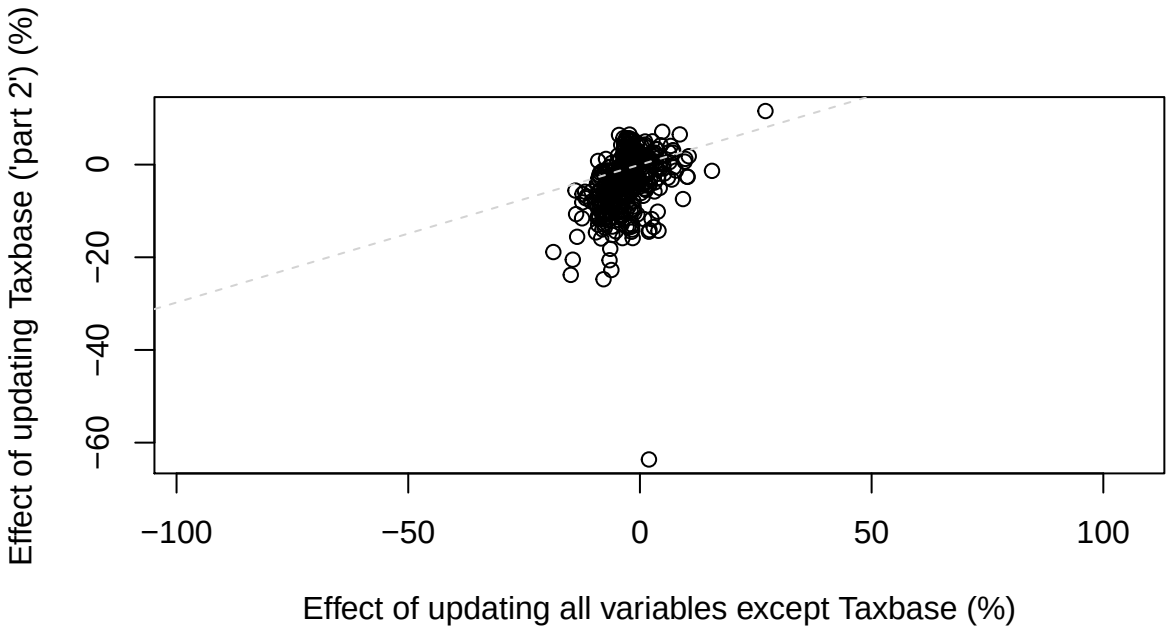

Figure 8: Effect of updating Taxbase with 'part 2' values compared to effects of updating all other variables.